

# Cluster production in AMD model

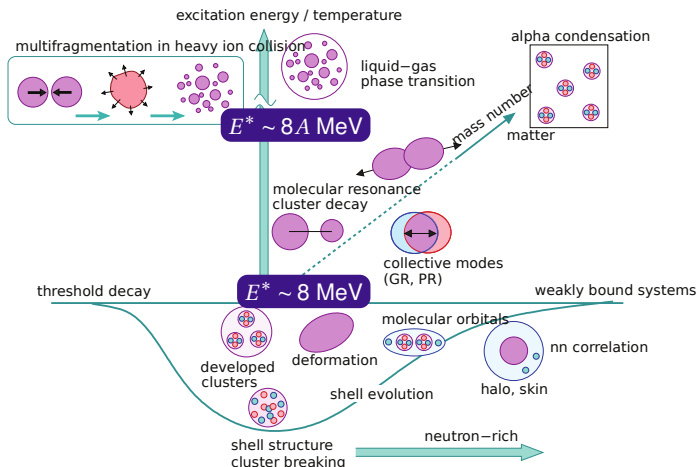
Akira Ono

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The 2017 ICNT Program at FRIB: Extracting Bulk Properties of Neutron-Rich Matter  
with Transport Models in Bayesian Perspective,  
FRIB-MSU, East Lansing, Michigan, USA March 22 - April 12, 2017

# Clustering phenomena in excited states of nuclear systems

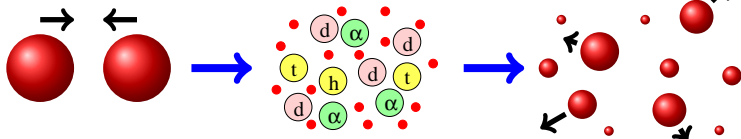
$E^* \sim 80A \text{ MeV}$  Gas of clusters at higher energies



Kanada-En'yo, Kimura, Ono, Prog. Theor. Exp. Phys. 2012 01A202 (2012)

# Importance of clusters in heavy-ion collisions

Collisions of two nuclei (e.g., Xe + Sn at 50 MeV/nucleon,  $b \approx 0$ )



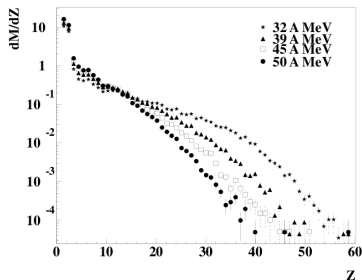
Light-cluster correlations

## Partitioning of protons

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	$\approx 10\%$	21%
$\alpha$	$\approx 20\%$	20%
d, t, $^3\text{He}$	$\approx 10\%$	40%
$A > 4$	$\approx 60\%$	18%

INDRA data, Hudan et al., PRC67 (2003) 064613.

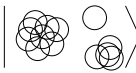
FOPi data, Reisdorf et al., NPA 848 (2010) 366.



Light-cluster correlations may be important at relatively early times.

# Antisymmetrized Molecular Dynamics (very basic version)

## AMD wave function



$$|\Phi(Z)\rangle = \det_{ij} \left[ \exp \left\{ -v \left( \mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{v}} \right)^2 \right\} \chi_{\alpha_i}(j) \right]$$

$$\mathbf{Z}_i = \sqrt{v} \mathbf{D}_i + \frac{i}{2\hbar\sqrt{v}} \mathbf{K}_i$$

$v$ : Width parameter =  $(2.5 \text{ fm})^{-2}$

$\chi_{\alpha_i}$ : Spin-isospin states =  $p \uparrow, p \downarrow, n \uparrow, n \downarrow$

## Equation of motion for the wave packet centroids $Z$

$$\frac{d}{dt} \mathbf{Z}_i = \{ \mathbf{Z}_i, \mathcal{H} \}_{\text{PB}} + (\text{NN collisions})$$

### $\{ \mathbf{Z}_i, \mathcal{H} \}_{\text{PB}}$ : Motion in the mean field

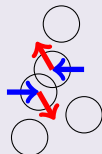
$$\mathcal{H} = \frac{\langle \Phi(Z) | H | \Phi(Z) \rangle}{\langle \Phi(Z) | \Phi(Z) \rangle} + (\text{c.m. correction})$$

$H$ : Effective interaction (e.g. Skyrme force)

### NN collisions

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i)$$

- $|V|^2$  or  $\sigma_{NN}$  (in medium)
- Pauli blocking



Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

## Wigner function for the AMD wave function

$$f_{\alpha}(\mathbf{r}, \mathbf{p}) = 8 \sum_{i \in \alpha} \sum_{j \in \alpha} e^{-2v(\mathbf{r} - \mathbf{R}_{ij})^2} e^{-(\mathbf{p} - \mathbf{P}_{ij})^2 / 2\hbar^2 v} B_{ij} B_{ji}^{-1}, \quad \alpha = p \uparrow, p \downarrow, n \uparrow, n \downarrow$$

$$\mathbf{R}_{ij} = \frac{1}{2\sqrt{v}}(\mathbf{Z}_i^* + \mathbf{Z}_j), \quad \mathbf{P}_{ij} = i\hbar\sqrt{v}(\mathbf{Z}_i^* - \mathbf{Z}_j), \quad B_{ij} = e^{-\frac{1}{2}(\mathbf{Z}_i^* - \mathbf{Z}_j)^2}$$

## Equation of motion for the wave packet centroids $Z$

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### $\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$ : Motion in the mean field

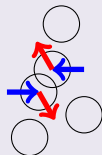
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- Pauli blocking



Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

## Skyrme force

$$\begin{aligned}
 v_{ij} = & t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1 + x_1 P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2 + \mathbf{k}^2\delta(\mathbf{r})] & \mathbf{r} = \mathbf{r}_i - \mathbf{r}_j \\
 & + t_2(1 + x_2 P_\sigma)\mathbf{k} \cdot \delta(\mathbf{r})\mathbf{k} + t_3(1 + x_3 P_\sigma)[\rho(\mathbf{r}_i)]^\alpha \delta(\mathbf{r}) & \mathbf{k} = \frac{1}{2\hbar}(\mathbf{p}_i - \mathbf{p}_j)
 \end{aligned}$$

Spatial integration of the potential energy density which is a function of several kind of densities.

$$\langle V \rangle = \int \mathcal{V}(\rho(\mathbf{r}), \tau(\mathbf{r}), \Delta\rho(\mathbf{r}), \mathbf{j}(\mathbf{r})) d\mathbf{r} \quad \sim A^2 \times \text{Volume}$$

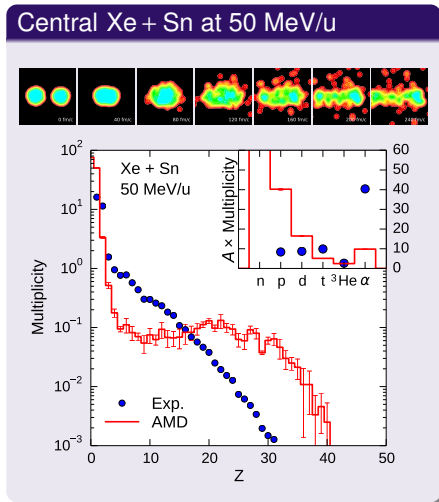
$$\rho_\alpha(\mathbf{r}) = \int f_\alpha(\mathbf{r}, \mathbf{p}) \frac{d\mathbf{p}}{(2\pi\hbar)^3} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{i \in \alpha} \sum_{j \in \alpha} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B_{ji}^{-1}, \quad \mathbf{R}_{ij} = \frac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^* + \mathbf{Z}_j)$$

$$\mathbf{j}_\alpha(\mathbf{r}) = \int \frac{\mathbf{p}}{M} f_\alpha(\mathbf{r}, \mathbf{p}) \frac{d\mathbf{p}}{(2\pi\hbar)^3} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{i \in \alpha} \sum_{j \in \alpha} \frac{\mathbf{p}_{ij}}{M} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B_{ji}^{-1}, \quad \mathbf{p}_{ij} = i\hbar\sqrt{\nu}(\mathbf{Z}_i^* - \mathbf{Z}_j)$$

$$\tau_\alpha(\mathbf{r}) = \int \frac{\mathbf{p}^2}{M^2} f_\alpha(\mathbf{r}, \mathbf{p}) \frac{d\mathbf{p}}{(2\pi\hbar)^3} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{i \in \alpha} \sum_{j \in \alpha} \frac{\mathbf{p}_{ij}^2 + 3\hbar^2\nu}{M^2} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B_{ji}^{-1}$$

# Failure of fragmentation and cluster production

AMD with usual NN collisions (very basic version)



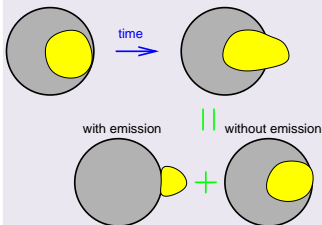
Partitioning of protons  
(experimental data)

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	$\approx 10\%$	21%
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INDRA data, Hudan et al., PRC67 (2003) 064613.

FOPi data, Reisdorf et al., NPA 848 (2010) 366.

## Two directions of extension of AMD

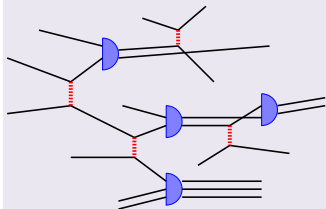


Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the **single-particle motion**.

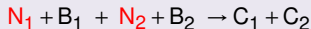
$$\frac{d}{dt}Z = \{Z, \mathcal{H}\}_{\text{PB}} + (\text{NN Collision}) \\ + (\text{W.P. Splitting}) + (\text{E. Conservation})$$

Ono, Hudan, Chibihi, Frankland, PRC66 (2002) 014603

Ono and Horiuchi, PPNP53 (2004) 501



At each two-nucleon collision, **cluster formation** is considered for the final state.



$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \text{CC} | V_{NN} | \text{NBNB} \rangle|^2 \delta(\mathcal{H} - E)$$

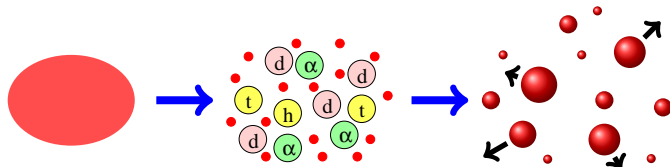
Ono, J. Phys. Conf. Ser. 420 (2013) 012103

Ikeno, Ono et al., PRC 93 (2016) 044612



# Interacting and reacting clusters in heavy-ion collisions

- The system may be composed of many clusters.
- Clusters are not only created but also broken by reactions.



Transport models with clusters

+decay

I want a transport model which can describe the dynamics for a sufficiently long time (e.g.,  $t \sim 1000 \text{ fm}/c$ ).

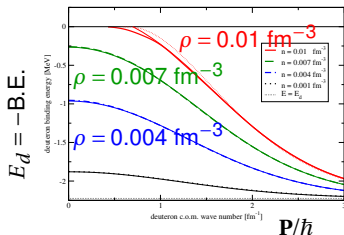
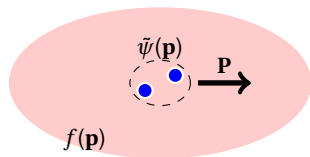
The decays of the excited fragments at the end of the dynamical calculation are calculated by a statistical decay code.

- $n + p + X \leftrightarrow d + X'$
- $d + n + X \leftrightarrow t + X'$
- $d + p + X \leftrightarrow h + X'$
- $t + p + X \leftrightarrow \alpha + X'$
- $h + n + X \leftrightarrow \alpha + X'$
- $d + d + X \leftrightarrow \alpha + X'$
- $2n + p + X \leftrightarrow t + X'$
- $n + 2p + X \leftrightarrow h + X'$
- $d + n + p + X \leftrightarrow \alpha + X'$
- $2n + 2p + X \leftrightarrow \alpha + X'$
- $d + d \leftrightarrow p + t$
- $d + d \leftrightarrow n + h$
- $p + t \leftrightarrow n + h$
- $d + t \leftrightarrow n + \alpha$
- $d + h \leftrightarrow p + \alpha$
- $d + t \leftrightarrow 2n + h$
- $d + h \leftrightarrow 2p + t$
- $d + \alpha \leftrightarrow t + h$

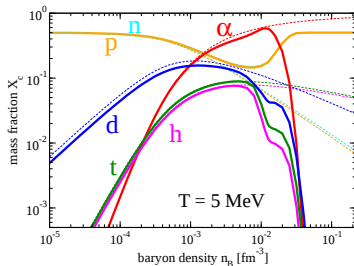
# A cluster in medium & Clusterized nuclear matter

## Equation for a deuteron in uncorrelated medium

$$\left[ e\left(\frac{1}{2}\mathbf{P} + \mathbf{p}\right) + e\left(\frac{1}{2}\mathbf{P} - \mathbf{p}\right) \right] \tilde{\psi}(\mathbf{p}) + \left[ 1 - f\left(\frac{1}{2}\mathbf{P} + \mathbf{p}\right) - f\left(\frac{1}{2}\mathbf{P} - \mathbf{p}\right) \right] \int \frac{d\mathbf{p}'}{(2\pi)^3} \langle \mathbf{p} | \nu | \mathbf{p}' \rangle \tilde{\psi}(\mathbf{p}') = E \tilde{\psi}(\mathbf{p})$$

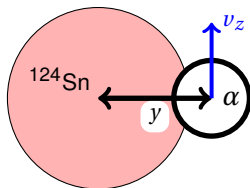


Momentum ( $\mathbf{P}$ ) dependence of B.E.  
Röpke, NPA867 (2011) 66.



QS for symmetric nuclear matter  
Röpke, PRC 92 (2015) 054001.

# A cluster put into a nucleus in AMD



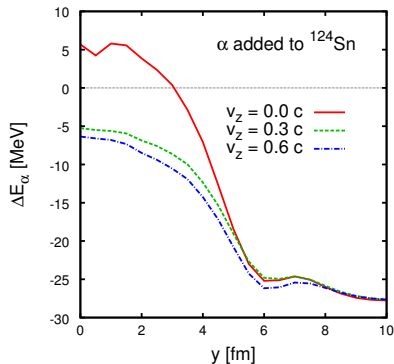
$\alpha$  cluster  $|\alpha, \mathbf{Z}\rangle$  : Four wave packets with different spins and isospins at the same phase space point  $\mathbf{Z}$ .

$$E_\alpha : \mathcal{A} |\alpha, \mathbf{Z}\rangle |^{124}\text{Sn}\rangle$$

$$E_N : \mathcal{A} |\mathbf{Z}\rangle |^{124}\text{Sn}\rangle \quad (N = p \uparrow, p \downarrow, n \uparrow, n \downarrow)$$

$$-B_\alpha = \Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$$

(Energies are defined relative to  $|^{124}\text{Sn}\rangle$ .)

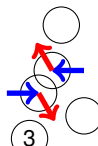


$$\frac{\text{Re}\mathbf{Z}}{\sqrt{v}} = (0, y, 0), \quad \frac{2\hbar\sqrt{v}\text{Im}\mathbf{Z}}{M} = (0, 0, v_z)$$

- Distance from the center:  $y$   
 $\approx$  Dependence on density
- Dependence on  $P_\alpha = M_\alpha v_z$
- Due to the density dependence of the Skyrme force, the interaction between nucleons in the  $\alpha$  cluster is weakened in the nucleus.

Energy is OK, but dynamics is ...

# NN collisions without or with cluster correlations

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i)$$


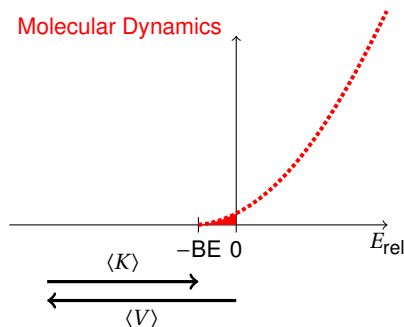
In the usual way of NN collision, only the two wave packets are changed.

$$\{ |\Psi_f\rangle \} = \{ |\varphi_{k_1}(1)\varphi_{k_2}(2)\Psi(3,4,\dots)\rangle \}$$

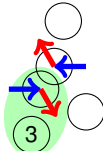
(ignoring antisymmetrization for simplicity of presentation.)

Phase space or the density of states for two nucleon system

Molecular Dynamics



# NN collisions without or with cluster correlations

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(ignoring antisymmetrization for simplicity of presentation.)

## Extension for cluster correlations

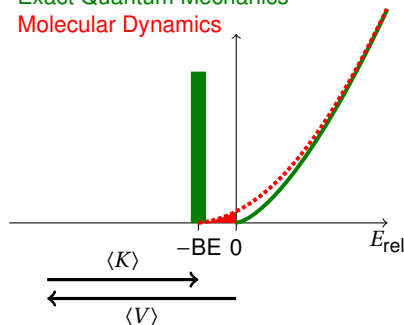
Include correlated states in the set of the final states of each NN collision.

$$\{ |\Psi_f\rangle \} \ni |\varphi_{k_1}(1)\psi_d(2,3)\Psi(4,\dots)\rangle, \dots$$

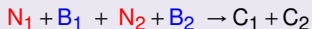
Phase space or the density of states for two nucleon system

Exact Quantum Mechanics

Molecular Dynamics



# NN collisions with cluster correlations



- $N_1, N_2$  : Colliding nucleons
- $B_1, B_2$  : Spectator nucleons/clusters
- $C_1, C_2$  :  $N, (2N), (3N), (4N)$  (up to  $\alpha$  cluster)

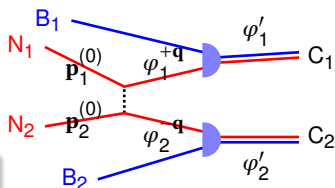
## Transition probability

$$W(\text{NBNB} \rightarrow \text{CC}) = \frac{2\pi}{\hbar} |\langle \text{CC} | V | \text{NBNB} \rangle|^2 \delta(E_f - E_i)$$

$$v d\sigma \propto |\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\text{rel}}^2 dp_{\text{rel}} d\Omega$$

$|M|^2 = |\langle \text{NN} | V | \text{NN} \rangle|^2$ : Matrix elements of NN scattering  
 $\Leftarrow (d\sigma/d\Omega)_{\text{NN}}$  in medium (or in free space)

Similar to Danielewicz et al.,  
 NPA533 (1991) 712.



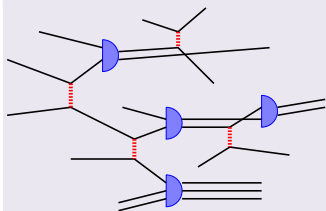
$$\mathbf{p}_{\text{rel}} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\text{rel}} \hat{\Omega}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

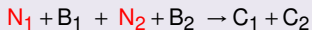
$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q} \cdot \mathbf{r}_{N_1}) \varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q} \cdot \mathbf{r}_{N_2}) \varphi_2^{(0)}$$

## NN collisions with cluster correlations (more explanations)



For each NN collision, cluster formation is considered.



$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle CC | V_{NN} | NBNB \rangle|^2 \delta(E_f - E_i)$$

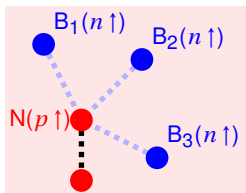
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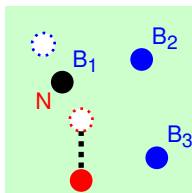
- We always have a Slater determinant of nucleon wave packets. A cluster in the final states is represented by placing wave packets at the same phase space point.
- Consequently the processes such as  $d + X \rightarrow n + p + X'$  and  $d + X \rightarrow d + X'$  are automatically taken into account.
- No parameters have been introduced to adjust individual reactions. But the cluster formation may be artificially weakened (nnchange\_gamma).
- There are many possibilities to form clusters in the final states. Non-orthogonality of the final states should be carefully handled.

# Construction of Final States

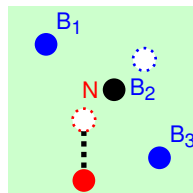
Clusters (in the final states) are assumed to have  $(0s)^N$  configuration.



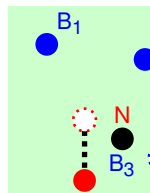
$|\Phi^{\mathbf{q}}\rangle$   
After  $\mathbf{p}^{(0)} \rightarrow \mathbf{p}^{(0)} + \mathbf{q}$



$|\Phi'_1\rangle$   
 $N + B_1 \rightarrow C_1$



$|\Phi'_2\rangle$   
 $N + B_2 \rightarrow C_2$



$|\Phi'_3\rangle$   
 $N + B_3 \rightarrow C_3$

Final states are not orthogonal:  $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

The probability of cluster formation with one of B's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N_{ij}^{-1} \langle \Phi'_j|, \quad P = \langle \Phi^{\mathbf{q}} | \hat{P} | \Phi^{\mathbf{q}} \rangle \neq \sum_i |\langle \Phi'_i | \Phi^{\mathbf{q}} \rangle|^2$$

- $\left\{ \begin{array}{l} P \\ 1 - P \end{array} \right. \Rightarrow$  Choose one of the candidates and make a cluster.
- $\left\{ \begin{array}{l} P \\ 1 - P \end{array} \right. \Rightarrow$  Don't make a cluster (with any  $n\uparrow$ ).



# An algorithm to decide cluster formation

decide to do a collision based on  $(d\sigma/d\Omega)_{NN}$

$C = N$

do for **species** in  $p \uparrow, p \downarrow, n \uparrow, n \downarrow$  (in a random order)

$P$  = probability that  $C$  forms a cluster with a nucleon of **species**

- taking care of the non-orthogonality
- taking care of the  $p_{rel}$ -dependence of the phase space factors and the overlap probabilities

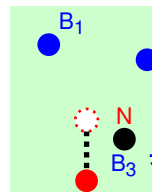
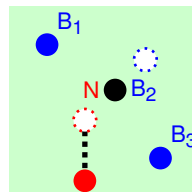
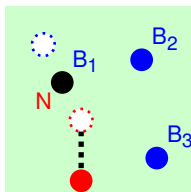
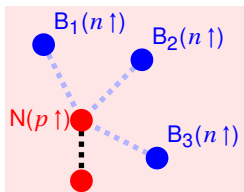
if  $\text{rand}() < P$  then

choose a nucleon  $B$  of **species**

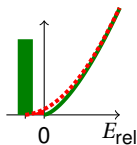
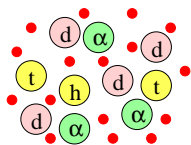
$C = C + B$  ! put the wave packets at the same phase space point

endif

enddo



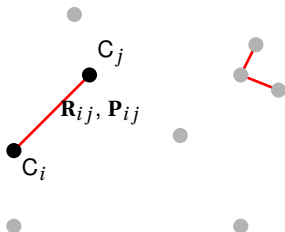
# Correlations to bind several clusters



Clusters may form a loosely bound state.

e.g.,  ${}^7\text{Li} = \alpha + t - 2.5 \text{ MeV}$

Need more probability of  $|\alpha + t\rangle \rightarrow |{}^7\text{Li}\rangle$



**Step 1** Clusters (and nucleons)  $C_i$  and  $C_j$  are *linked*,

- if  $C_i$  is one of the **3** clusters closest to  $C_j$ , and  $(i \leftrightarrow j)$ ,
- and if the distance is **1 fm**  $< |\mathbf{R}_{ij}| < 7 \text{ fm}$ ,
- and if they are slowly moving away,  $\mathbf{P}_{ij}^2/2\mu_{ij} < \mathbf{10 MeV}$  and  $\mathbf{R}_{ij} \cdot \mathbf{P}_{ij} > 0$ .

**Step 2** Linked clusters (CC) are identified.

Following steps are taken only for CC with mass number **6**  $\leq A \leq 9$  or **19**  $\leq A \leq 23$ .

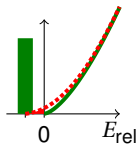
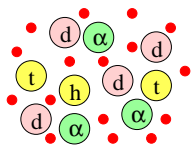
**Step 3** *Transition* of the internal state of CC by eliminating the (radial component of) internal momentum

$$\mathbf{P}_i \rightarrow 0 \quad \text{for } i \in \text{CC in the c.m. of CC}$$

with some care of the momentum conservation.

**Next** *Energy conservation.*

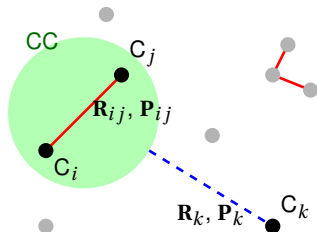
# Correlations to bind several clusters



Clusters may form a loosely bound state.

e.g.,  ${}^7\text{Li} = \alpha + t - 2.5 \text{ MeV}$

Need more probability of  $|\alpha + t\rangle \rightarrow |{}^7\text{Li}\rangle$



**Step 4** Search a third particle for E-conservation

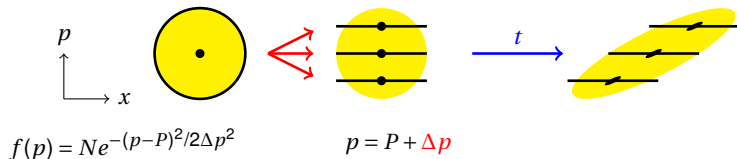
- A cluster  $C_k$  is selected, depending on the distance and momentum ( $|\mathbf{R}_k|$  and  $|\mathbf{P}_k|$ ) relative to CC.
- If the selected  $C_k$  already belongs to a  $CC'$ , this whole  $CC'$  is treated as the third particle for E-conservation.

**Step 5** Scale the radial component of the relative momentum between CC and  $C_k$  for the total energy conservation.

$$\mathbf{P}_k = \mathbf{P}_{k\parallel} + \mathbf{P}_{k\perp} \rightarrow \beta \mathbf{P}_{k\parallel} + \mathbf{P}_{k\perp}$$

## Transition from a wave packet to a plane wave

Each wave packet has a momentum width. E.g., it is an important part of the Fermi motion.

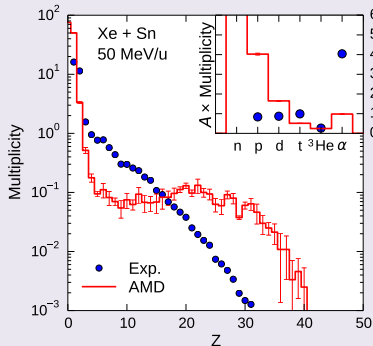
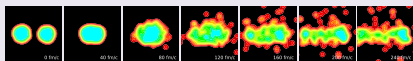


The momentum fluctuation  $\Delta p$  is given to a wave packet when it is '**emitted**', following [Ono and Horiuchi, PRC53 \(1996\) 845](#) [a simple version of wp splitting].

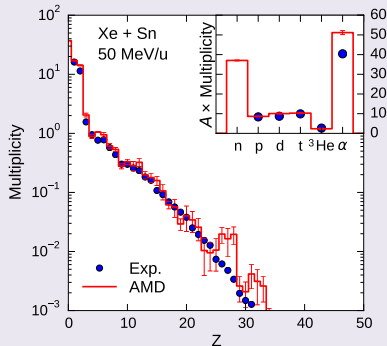
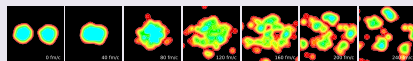
- For a formed cluster, the momentum fluctuation is given to its center-of-mass motion.
- Total momentum and energy conservation.
- A particle is regarded as '**emitted**' when there is no other particles around it in phase space within the radius  $(\Delta r, \Delta v) = (3.5 \text{ fm}, 0.25c)$ .
- Consistency with the method of the zero-point energy correction.

# Effect of cluster correlations: central Xe + Sn at 50 MeV/u

## Without clusters

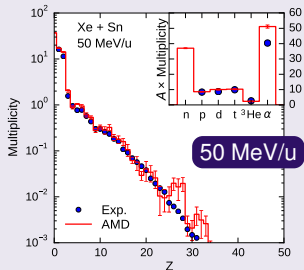
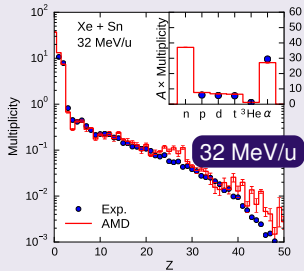


## With clusters

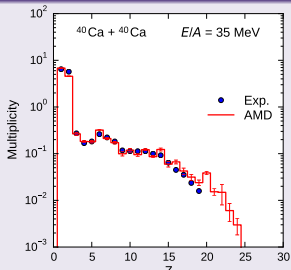


# Results for multifragmentation in central collisions

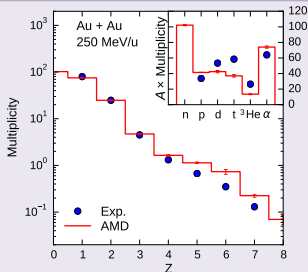
## Xe + Sn



## Ca + Ca at 35 MeV/u



## Au + Au at 250 MeV/u

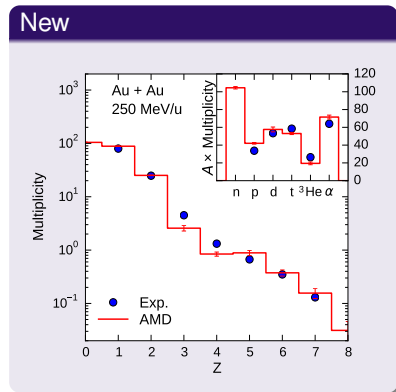
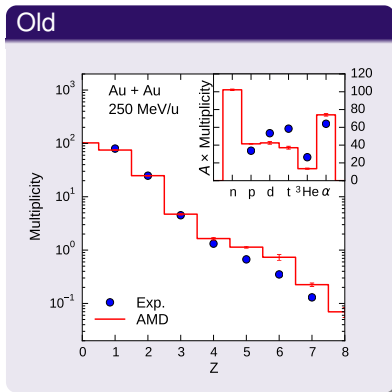
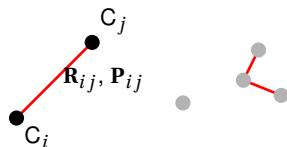


Data: Hudan et al., PRC 67 (2003) 064613.  
 Hagel et al., PRC 50 (1994) 2017.  
 Reisdorf et al., NPA 848 (2010) 366.

# An improvement related to $\alpha$ clusters

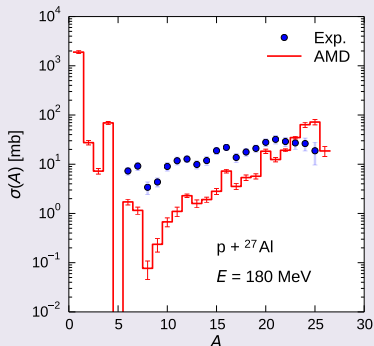
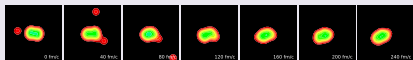
Changes:

- Link two clusters only if at least one of them is  $\alpha$ .
- Don't produce  $\alpha$  clusters at high densities  $\rho > \rho_0$ .



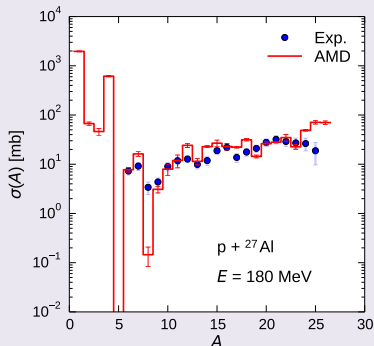
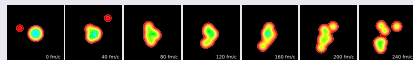
# Effect of cluster correlations: $p + \text{Al}$ at 180 MeV

## Without clusters



Data: K. Kwiatkowski et al., PRL50(1983)1648.

## With clusters

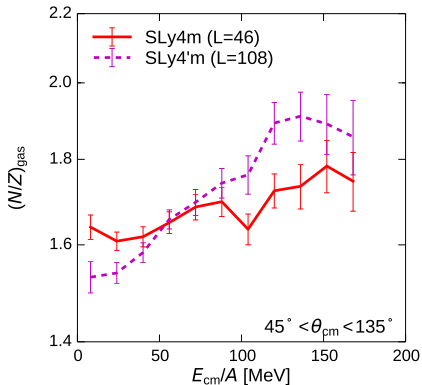


The result is sensitive to the inmedium two-nucleon cross sections.

$$\sigma_{\text{NN}} = \sigma_0 \tanh(\sigma_{\text{free}}/\sigma_0), \quad \sigma_0 = y\rho^{-2/3}, \quad y = 4. \quad \text{c.f. Coupland et al., PRC84(2011)054603}$$

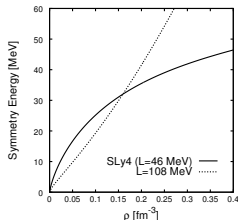
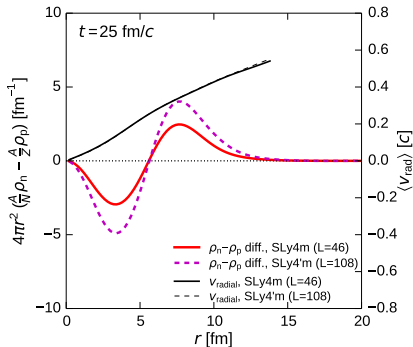


# N/Z Ratio in $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/u (AMD with clusters)

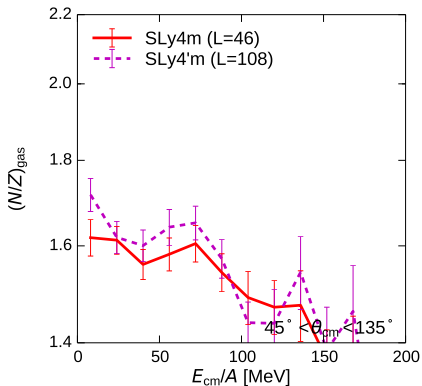


$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

N/Z of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.

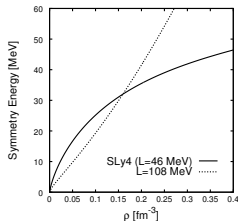
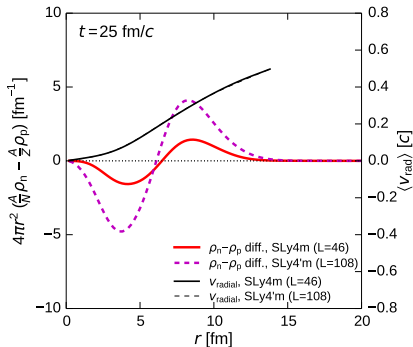


# N/Z Ratio in $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/u (AMD without clusters)



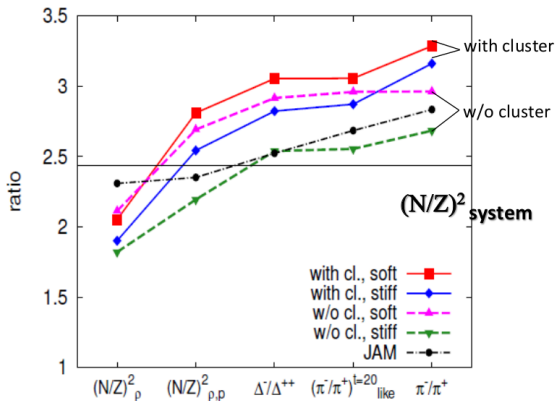
$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(v) + Y_d(v) + 2Y_t(v) + Y_h(v) + 2Y_\alpha(v)}{Y_p(v) + Y_d(v) + Y_t(v) + 2Y_h(v) + 2Y_\alpha(v)}$$

N/Z of spectrum of emitted particles is NOT similar to the neutron-proton density difference at the compression stage.



# Summary of ratios, for $^{132}\text{Sn} + ^{124}\text{Sn}$ at 300 MeV/nucleon

Ikeno's talk; Ikeno, Ono, Nara, Ohnishi, PRC 93 (2016) 044612



## Recent developments of AMD

- Cluster correlations in the final states of NN collisions
- Binding of several clusters (production of Li, Be, . . . )
- Treatment of the wave-packet momentum width
- Test particles sampled from  $f(\mathbf{r}, \mathbf{p})$  of AMD
  - Comparison with other models
  - Combining with another model (pion production)
  - Improvement of NN collision procedure